

[SUNY BING 8/12/00]

hep-ph/0106137

**ON TESTING FOR NEW COUPLINGS IN TOP QUARK DECAY<sup>1</sup>**Charles A. Nelson<sup>2</sup>*Department of Physics, State University of New York at Binghamton**Binghamton, N.Y. 13902-6016***Abstract**

To quantitatively assay future measurements of competing observables in  $t \rightarrow W^+b$  decay, we consider the  $g_{V-A}$  coupling values of the helicity decay parameters versus those for “ $(V - A) + \text{Single Additional Lorentz Structures}$ ”. There are 2 dynamical phase-type ambiguities  $(S + P)$  and  $(f_M + f_E)$ . Associated with the latter  $(f_M + f_E)$  ambiguity, there are 3 very interesting numerical puzzles at the mil level. This evidence for the presence of tensorial couplings in  $t \rightarrow W^+b$  decay is a consequence of the empirical value of  $m_W/m_t$  and the small, but non-zero, ratio  $m_b/m_t$ . Measurement of the sign of  $|\eta_L| = 0.46(\text{SM})$  due to the large interference between the W longitudinal/transverse amplitudes would exclude such tensorial couplings. Similarly, sufficiently precise measurements of both  $\eta_L$  and  $\eta_L'$  could resolve the analogous dynamical ambiguity in the case of a partially-hidden  $T$ -violation associated with the additional  $f_M + f_E$  coupling.

---

<sup>1</sup>For Proceedings of “IVth Rencontres du Vietnam” .

<sup>2</sup>Electronic address: cnelson @ binghamton.edu

# 1 Introduction

In physics at the highest available energies, it is always important to exploit simple reactions and decays so as to search for new forces, for new dynamics, and for discrete symmetry violations. Because the  $t$ -quark weakly decays before hadronization effects are significant, and because of the large  $t$ -quark mass,  $t$ -quark decay can be an extremely useful tool for such fundamental searches. Initial tests of the Lorentz structure and of symmetry properties of  $t \rightarrow W^+ b$  decay will be carried out at the Tevatron[1], but the more precise measurements will be possible at the CERN LHC [2] and at a NLC [2].

It is important to be able to quantitatively assay future measurements of competing observables consistent with the standard model (SM) prediction of only a  $g_{V-A}$  coupling and only its associated discrete symmetry violations. For this purpose, without consideration of possible explicit  $T$ -violation, in Ref.[3] plots were given of the values of the helicity parameters in terms of a “ $(V - A) + \text{Additional Lorentz Structure}$ ” versus effective-mass scales for new physics,  $\Lambda_i$ , associated with each additional Lorentz structure. Recently in Ref.[4], to assay future measurements of helicity parameters in regard to  $T$ -violation, the effects of possible explicit  $T$ -violation were reported. In the present formulation, by “explicit  $T$ -violation”, we mean an additional complex-coupling,  $g_i/2\Lambda_i$  or  $g_i$ , associated with a specific single additional Lorentz structure,  $i = S, P, S \pm P, \dots$ . In effective field theory,  $\Lambda_i$ , is the scale at which new particle thresholds or new dynamics are expected to occur;  $\Lambda_i$  can also be interpreted as a measure of a top quark compositeness/condensate scale. In measurement of some of the helicity parameters, the LHC should be sensitive to  $\sim 3\%$  and the Tevatron in a “Run 2B” to perhaps the  $\sim 10\%$  level (“ideal statistical error levels”) [5].

## 2 Testing for the Complete Lorentz Structure in Absence of Explicit $T$ -Violation

A complete measurement of on-shell properties of the  $t \rightarrow W^+ b$  decay mode will have been accomplished when the 4 moduli are determined and any 3 of the relative phases of the helicity amplitudes  $A(\lambda_{W^+}, \lambda_b)$ . The helicity parameters appear directly in various polarization and spin-correlation functions such as those obtained in Ref.[5]. Since the helicity parameters appear directly in the various polarization and spin-correlation functions, it is clearly more model independent to simply measure them rather than to set limits on an “ad hoc” set of additional coupling constants [3].

In the plots in Refs.[3,4], the values of the helicity parameters are given in terms of a “ $(V-A) + \text{Single Additional Lorentz Structure}$ ”. Generically, in the case of no explicit  $T$ -violation, we denote these additional couplings by

$$g_{Total} \equiv g_L + g_X \quad (1)$$

$$X = \begin{cases} X_c = \text{chiral} = \{V + A, S \pm P, f_M \pm f_E\} \\ X_{nc} = \text{non-chiral} = \{V, A, S, P, f_M, f_E\}. \end{cases}$$

For  $t \rightarrow W^+ b$ , the most general Lorentz coupling is  $W_\mu^* J_{bt}^\mu = W_\mu^* \bar{u}_b(p) \Gamma^\mu u_t(k)$  where  $k_t = q_W + p_b$ , and

$$\begin{aligned} \Gamma_V^\mu = & g_V \gamma^\mu + \frac{f_M}{2\Lambda} \iota \sigma^{\mu\nu} (k-p)_\nu + \frac{g_{S^-}}{2\Lambda} (k-p)^\mu \\ & + \frac{g_S}{2\Lambda} (k+p)^\mu + \frac{g_{T^+}}{2\Lambda} \iota \sigma^{\mu\nu} (k+p)_\nu \end{aligned} \quad (2)$$

$$\begin{aligned} \Gamma_A^\mu = & g_A \gamma^\mu \gamma_5 + \frac{f_E}{2\Lambda} \iota \sigma^{\mu\nu} (k-p)_\nu \gamma_5 + \frac{g_{P^-}}{2\Lambda} (k-p)^\mu \gamma_5 \\ & + \frac{g_P}{2\Lambda} (k+p)^\mu \gamma_5 + \frac{g_{T_5^+}}{2\Lambda} \iota \sigma^{\mu\nu} (k+p)_\nu \gamma_5 \end{aligned} \quad (3)$$

For  $g_L = 1$  units with  $g_i = 1$ , the nominal size of  $\Lambda_i$  is  $\frac{m_t}{2} = 88\text{GeV}$ , see [3]. In the SM, the EW energy-scale is set from the Higgs-field vacuum-expectation-value by the parameter  $v = \sqrt{-\mu^2/|\lambda|} = \sqrt{2}\langle 0|\phi|0\rangle \sim 246\text{GeV}$ . Lorentz equivalence theorems for these couplings are treated in Ref.[3]. Explicit expressions for the  $A(\lambda_{W^+}, \lambda_b)$  in the case of these additional Lorentz structures are given in Ref. [5].

In Table 1 in the top line are the standard model expectations for the numerical values of the helicity amplitudes  $A(\lambda_{W^+}, \lambda_b)$  for  $t \rightarrow W^+b$  decay in  $g_L = 1$  units. The input values are  $m_t = 175\text{GeV}$ ,  $m_W = 80.35\text{GeV}$ ,  $m_b = 4.5\text{GeV}$ . The  $\lambda_b = 1/2$  b-quark helicity amplitudes would vanish if  $m_b$  were zero. For this reason, if the SM is correct, one expects that the  $A(0, -1/2)$  and  $A(-1, -1/2)$  moduli and relative phase  $\beta_L$  will be the first quantities to be determined. The  $\lambda_b = 1/2$  moduli are factors of 30 and 100 smaller in the SM. Throughout this moduli-phase analysis of top decays, intrinsic and relative signs of the helicity amplitudes are specified in accordance with the standard Jacob-Wick phase convention.

Versus predictions based on the SM, two dynamical phase-type ambiguities were found by investigation of the effects of a single additional “chiral” coupling  $g_i$  on the three moduli parameters  $\sigma = P(W_L) - P(W_T)$ ,  $\xi = P(b_L) - P(b_R)$ , and  $\zeta = \frac{1}{\Gamma}(\Gamma_L^{b_L-b_R} - \Gamma_T^{b_L-b_R})$ . The quantities

$$P(W_L) = \text{Probability } W^+ \text{ is longitudinally polarized, } \lambda_{W^+} = 0$$

$$P(b_L) = \text{Probability } b \text{ is left-handed, } \lambda_b = -1/2$$

In the SM, the final  $W$  boson should be 70% longitudinally polarized and the b-quark should be almost completely left-handed polarized.

(1) For an additional  $S + P$  coupling with  $\Lambda_{S+P} \sim -34.5\text{GeV}$  the values of  $(\sigma, \xi, \zeta)$  and of the partial width  $\Gamma$  are about the same as the SM prediction. Table 1 shows that this ambiguity

occurs because the sign of the  $A_X(0, -\frac{1}{2})$  amplitude for  $g_L + g_X$  is opposite to that of the SM's amplitude. (2) For an additional  $f_M + f_E$  coupling with  $\Lambda_{f_M+f_E} \sim 53\text{GeV}$  the values of  $(\sigma, \xi, \zeta)$  are also about the same as the SM prediction. In this case, the partial width  $\Gamma$  is about half that of the SM due to destructive interference. (3) From consideration of Table 1, a third (non-dynamical) phase ambiguity can be constructed by making an arbitrary sign-flip in the  $b_L$  amplitudes, with no corresponding sign changes in the  $b_R$  amplitudes. Its exclusion, as well as determination of the 2 remaining independent relative phases necessary for a complete amplitude measurement will require direct empirical information about the  $b_R$ -amplitudes such as from a  $\Lambda_b$  polarimetry measurement [5] of the  $b$ -polarimetry interference parameters. Such measurements will be difficult unless certain non-SM couplings occur: non-chiral couplings like  $V$  or  $A$ ,  $f_M$  or  $f_E$  (for  $\epsilon_+$ ),  $S$  or  $P$  (for  $\kappa_0$ ) can produce large effects[3]. Two dimensional plots of the type  $(\epsilon_{+,-}, \eta_L)$  and  $(\kappa_{0,1}, \eta_L)$ , and of their primed counterparts, have the useful property that the unitarity limit is a circle of radius 0.5 centered on the origin.

### 3 Remarks on the Dynamical Phase-type Ambiguities

Due the dominance of the L-handed amplitudes in the SM, the occurrence of the two dynamical ambiguities [1] displayed in lower part of Table 1 is not surprising for these 3 chiral combinations only contribute to the L-handed b-quark amplitudes as  $m_b \rightarrow 0$ . Since pairwise the couplings are tensorially independent, the  $g_{V-A} + g_{S+P}$  &  $g_{V-A} + g_{f_M+f_E}$  mixtures can each be adjusted to reproduce, with opposite sign, the SM ratio of the two  $(\lambda_W = 0, -1)$  L-handed amplitudes.

However, in the case of the  $f_M + f_E$  phase-type ambiguity, from Table 1 there are 3 numerical puzzles at the mil level versus the SM values. In the upper part, the  $A_+(0, -1/2)$  amplitude for  $g_L +$

$g_{f_M+f_E}$  has about the same value in  $g_L = 1$  units, as the  $A_{SM}(-1, -1/2)$  amplitude in the SM. As  $m_b \rightarrow 0$ ,  $\frac{A_+(-1, -1/2)}{A_{SM}(0, -1/2)} \rightarrow \frac{m_t(m_t^2 - m_W^2)}{\sqrt{2}m_W(m_t^2 + m_W^2)} = 1.0038$ . The other numerical puzzle(s) is the occurrence in the lower part of the Table 1 of the same magnitude of the two R-handed b-quark amplitudes  $A_{New} = A_{g_L=1}/\sqrt{\Gamma}$  for the SM and for the case of  $g_L + g_{f_M+f_E}$ . Except for the differing partial width, by tuning the magnitude of L-handed amplitude ratio to that of the SM, the R-handed amplitude's moduli also become about those of the SM. With  $\Lambda_{f_M+f_E}$  determined as in Sec. 5, for the four  $A_{New}$  amplitudes  $|A_+| - |A_{SM}| \sim (m_b/m_t)^2 = 0.0007$  versus for instance  $|A_{SM}(\lambda_W, 1/2)| \propto m_b$ . Of course, the row with SM values is from a “theory” whereas the row of  $g_L + g_{f_M+f_E}$  values is not. Nevertheless, dynamical SSB and compositeness/condensate considerations do continue to stimulate interest [5] in additional tensorial  $f_M + f_E$  couplings. In Table 1, due to the additional  $f_M + f_E$  coupling, the net result is that it is the  $\mu = \lambda_{W^+} - \lambda_b = -1/2$  helicity amplitudes  $A_{New}$  which get an overall sign change. Fortunately, a sufficiently precise measurement of the sign of  $|\eta_L| = 0.46(\text{SM})$  due to the large interference between the W longitudinal/transverse amplitudes can resolve the  $V - A$  and  $f_M + f_E$  lines of this table. Measurement of the sign of the  $\eta_L \equiv \frac{1}{\Gamma}|A(-1, -\frac{1}{2})||A(0, -\frac{1}{2})| \cos \beta_L$  helicity parameter will determine the sign of  $\cos \beta_L$  where  $\beta_L$  is the relative phase of the two  $b_L$ -amplitudes.

## 4 Consequences of “Explicit $T$ -Violation”

The helicity formalism is based on the assumption of Lorentz invariance but not on any specific discrete symmetry property of the fundamental amplitudes, or couplings. For instance, for  $t \rightarrow W^+b$  and  $\bar{t} \rightarrow W^-\bar{b}$  in the case of  $T$ -invariance, the respective helicity amplitudes must be purely real,  $A^*(\lambda_{W^+}, \lambda_b) = A(\lambda_{W^+}, \lambda_b)$ ,  $B^*(\lambda_{W^-}, \lambda_{\bar{b}}) = B(\lambda_{W^-}, \lambda_{\bar{b}})$ . Consequently, all of the primed

helicity parameters [5,4] are zero..  $T$ -invariance will be violated if either (i) there is a fundamental violation of canonical “time reversal” invariance, or (ii) there are absorptive final-state interactions. In the SM, there are no such final-state interactions at the level of sensitivities considered in the present analysis. To assess future measurements of helicity parameters in regard to  $T$ -violation, Ref. [4] gives plots for the case of a single additional pure-imaginary coupling,  $ig_i/2\Lambda_i$  or  $ig_i$ , associated with a specific additional Lorentz structure. (i) An additional  $V - A$  type coupling with a complex phase versus the SM’s  $g_L$  is equivalent to an additional overall complex factor in the SM’s helicity amplitudes. This will effect the overall partial width  $\Gamma$ , but it doesn’t effect the other helicity parameters. For a single additional gauge-type coupling  $V, A$ , or  $V + A$ , there is not a significant signature in  $\eta_L'$  due to the  $T$ -violation “masking mechanism” associated with gauge-type couplings [5]. For example: for an additional pure imaginary  $g_R$  coupling plus  $g_L$ ,  $\eta_L' \sim m_b/m_t$ . So in [4], to test for the presence of  $T$ -violation due to additonal gauge type couplings, there are plots of the b-polarimetry interference parameters  $\epsilon_+'$  and  $\kappa_0'$ , and of the partial width for  $t \rightarrow W^+b$  versus pure-imaginary coupling constant  $ig_i$ . The respective peak magnitudes are  $\sim 0.23, \sim 0.35$  for the  $V + A$  coupling, and are  $\sim 0.16, \sim 0.25$  for the  $V, A$  couplings. (ii) Additional  $S \pm P, , f_M \pm f_E, S, P, f_M$ , or  $f_E$  couplings can lead to sizable signatures in the  $\eta_L' \equiv \frac{1}{\Gamma}|A(-1, -\frac{1}{2})||A(0, -\frac{1}{2})|\sin\beta_L$  helicity parameter for  $\Lambda_i \leq \sim 320GeV$ . There are also sizable induced effects (factors of  $\geq \sim 2$ ) of such additional couplings on the partial width for  $t \rightarrow W^+b$ . Ref.[4] also displays plots of the b-polarimetry interference parameters  $\epsilon_+'$  and  $\kappa_0'$  versus  $\Lambda_i$  for each of these couplings, except  $f_M + f_E$  which produces little effect on these 2 parameters. However, in most cases, such sizable signatures for explicit  $T$ -violation due to a single additional coupling can be more simply excluded by 10% precision measurement of the

probabilities  $P(W_L)$  and  $P(b_L)$ . The W-polarimetry interference parameters  $\eta$  and  $\omega$  can also be used as indirect tests, or to exclude such additional couplings.

## 5 Tests for “Partially Hidden $T$ -Violation”

It is possible that  $T$ -violation exists in the decay helicity amplitudes, but nevertheless does not significantly show up in the values of the moduli parameters. We call this “partially hidden  $T$ -violation” [4]. Based on the notion of a complex effective mass scale parameter  $\Lambda_X = |\Lambda_X| \exp(-i\theta)$  where  $\theta$  varies with the mass scale  $|\Lambda_X|$ , we exploit the dynamical phase-type ambiguities to construct two simple phenomenological models in which this happens. When  $\sin \theta \geq 0$ , the imaginary part of  $\Lambda_X$  could be interpreted as crudely describing a more detailed/realistic dynamics with a mean lifetime scale  $\Gamma_X \sim 2|\Lambda_X| \sin \theta$  of pair-produced particles at a production threshold  $Re[2\Lambda_X]$ . In the case of the  $f_M + f_E$  ambiguity, over the full  $\theta$  range, this construction preserves the magnitudes’ puzzle of Sec. 3. In Ref. [4] are plots of the signatures for a partially-hidden  $T$ -violation associated with the  $S + P$  and  $f_M + f_E$  phase-type ambiguities. Here we will discuss only the latter case. The additional  $f_M + f_E$  coupling  $g_{f_M+f_E}/2\Lambda_{f_M+f_E}$  now has an effective mass scale parameter  $\Lambda_{f_M+f_E} = |\Lambda_{f_M+f_E}| \exp(-i\theta)$  in which  $\theta$  varies with the mass scale  $|\Lambda_{f_M+f_E}|$  to maintain SM values in the massless b-quark limit for the moduli parameters  $P(W_L), P(b_L)$ , and  $\zeta$ . For  $X = f_M + f_E$ , we require  $\frac{|A_X(-1, -\frac{1}{2})|}{|A_X(0, -\frac{1}{2})|} = \frac{|A_L(-1, -\frac{1}{2})|}{|A_L(0, -\frac{1}{2})|}$  so for  $m_b = 0$  the relationship giving  $\theta(|\Lambda_{f_M+f_E}|)$  is  $\cos \theta \simeq \frac{m_t}{4\Lambda} (1 + (\frac{m_W}{m_t})^2)$  for  $52.9 GeV \leq |\Lambda_{f_M+f_E}| \leq \infty$  which correspond respectively to  $0 \leq \theta \leq \pm \frac{\pi}{2}$ . At the maximum of  $\eta_L'$ ,  $|\Lambda_{f_M+f_E}| \sim 63 GeV$ . Where  $\eta_L'$  has the maximum deviation, there is a zero in  $\eta_L, \eta, \omega$ . As the  $\Lambda$  scale increases, the  $\sim 2$  destructive interference effect in the partial width decreases monotonically. Sufficiently precise measurement of the W-interference



parameters  $\eta_L$  and  $\eta_L'$  can exclude such partially-hidden  $T$ -violation associated with either of the two dynamical phase-type ambiguities. [ This work was partially supported by U.S. Dept. of Energy Contract No. DE-FG 02-86ER40291.]

## References

- [1] F. Abe, et. al. (CDF collaboration), Phys. Rev. Lett. **74**, 2626(1995); S. Abachi, et. al. (D0 collaboration), Phys. Rev. Lett. **74**, 2632(1995).
- [2] ATLAS Technical Proposal, CERN/LHCC/94-43; CMS Technical Design Report, CERN-LHCC- 97-32. Reports on work for Next Linear Colliders by L. Maiani (CERN), A. Wagner (DESY), H. Sugawara (KEK) and J. Dorfan at ICHEP2000, Osaka.
- [3] C.A. Nelson and A.M. Cohen, Eur. Phys. J. **C8**, 393(1999).
- [4] C.A. Nelson and L.J. Adler, Eur. Phys. J. (in press), hep-ph/0007086; hep-ph/0006342.
- [5] C.A. Nelson, Kress, Lopes, McCauley, Phys. Rev. **D56**, 5928(1997); **D57**, 5923(1998).

## Table Captions

Table 1: For the ambiguous moduli points, numerical values of the associated helicity amplitudes  $A(\lambda_{W^+}, \lambda_b)$ . The values for the amplitudes are listed first in  $g_L = 1$  units, and second as  $A_{new} = A_{g_L=1}/\sqrt{\Gamma}$  which removes the effect of the differing partial width,  $\Gamma$  for  $t \rightarrow W^+b$ . [ $m_t = 175GeV$ ,  $m_W = 80.35GeV$ ,  $m_b = 4.5GeV$  ].

Table 1: Amplitudes at Ambiguous Moduli Points

	$A(0, -\frac{1}{2})$	$A(-1, -\frac{1}{2})$	$A(0, \frac{1}{2})$	$A(1, \frac{1}{2})$
$A_{g_L=1}$ in $g_L = 1$ units				
$V - A$	338	220	-2.33	-7.16
$S + P$	-338	220	-24.4	-7.16
$f_M + f_E$	220	-143	1.52	-4.67
$A_{New} = A_{g_L=1}/\sqrt{\Gamma}$				
$V - A$	0.84	0.54	-0.0058	-0.018
$S + P$	-0.84	0.54	-0.060	-0.018
$f_M + f_E$	0.84	-0.54	0.0058	-0.018

Table 2: Helicity Parameters at Ambiguous Moduli Points

	$\sigma$	$\xi$	$\zeta$	$\Gamma[GeV]$	
$V - A$	0.41	1.00	0.41	1.55GeV	
$S + P$	0.41	0.99	0.40	1.55GeV	
$f_M + f_E$	0.41	1.00	0.41	0.66GeV	
	$\eta$	$\omega$	$\eta_L$	$\kappa_o$	$\epsilon_+$
$V - A$	0.46	0.46	0.46	-0.005	-0.015
$S + P$	-0.45	-0.46	-0.46	0.05	0.015
$f_M + f_E$	-0.46	-0.46	-0.46	0.005	-0.015